Ph 129 : Mathematical Physics

Introduction

This course is officially based on "Mathematical Methods of Physics" by J. Mathews and R.L. Walker. Hereafter M. and W. In the notes, I will deviate quite a lot from this as I prefer to cover fewer topics than in M. and W. - but treat each chosen topic more deeply. M. and W. is an excellent a very readable introduction to a plethora of subjects - it is convenient to discuss the contents of my course by reference to their table of contents.

Chapter	1	Ordinary Differential Equations
	2	Infinite Series
	3	Evaluation of Integrals
	4	Integral Transforms
	5	Complex Variables
	6	Vectors and Matrices
	7	Special Functions
	8	Partial Differential Equations
	9	Eigenfunctions and Greens Functions
	10	Perturbation Theory
	11	Integral Equations.
	12	Calculus of Variations.
Appendix		Introduction to Complex Variables

. These first 12 chapters are rather classical and probably familiar to greater or lesser degree to the student. We will not cover, at all, chapters 2, 3 and the appendix : these are vital techniques and the student must read this material if he is at all unsure.

We will start with chapters 1 and 7 which are essentially methods of solving second order linear differential equations and use of special functions as an examples then we will cover an upgraded version of chapter 6 developing the ideas of function spaces and generalized functions. This can be illustrated by the various sets of special polynomials (Legendre, Hermite ..) found earlier. Given this technical expertise, we will then cover integral equations, Green's functions and partial differential equations (chapters 8 to 11).

Chapters 5 and 12 will not be covered.

• They are (relatively) specialized topics which will either be done in quantum physics (dispersion relations) or are probably already known. (calculus of variations / conformal maps). The interested student should read this material.

• The second half of the course covers a few specialized topics. M. and W. offer:

Chapter 13 : Numerical Methods.
14 : Probability and Statistics.
15 : Tensor Analysis.
16 : Group Theory.

We will cover chapters 13, 14, 16

• but leave 15 to a general relativity course.

We will also cover asymptotic expansions (part of chapter 3. in M. and W.) and, if time, other advanced ideas in complex variable theory. [entire functions]

A. Solution of Ordinary Differential Equations and applications to special Functions. (Practical Aspects)

A1 : Introduction and Omissions

we are discussing the equation:

$$F(x, y, y', \ldots y^{(n)}) = 0 \qquad (A1.1)$$

where $y = y(x)$ & we can define terms :

"ordinary" means y is a function of 1 variable only.

"order" $= n$ is highest derivative present

"degree" is power of highest derivative after equation has been rationalized in $y, y' \ldots y^{(n)}$. e.g. $y''' + x\sqrt{y} = 0$ has order 3 and degree 2.

"linear" implies equation can be written